

# Function Spaces with Pointwise Convergence Topology

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## Abstract

It is well known some properties are preserved by linear or uniform homeomorphisms between  $C_p(X)$  and  $C_p(Y)$ . Here  $C_p(X)$  (resp.,  $C_p^*(X)$ ) denotes the space of all continuous (resp., continuous and bounded) functions on  $X$  with the pointwise topology. For example  $\dim X = \dim Y$  if  $C_p(X)$  and  $C_p(Y)$  are linearly homeomorphic, see Pestov [4]. Later Gul'ko [2] extended this result by showing it is true for uniform homeomorphisms. In the present paper we investigate preservation of topological properties under continuous or uniformly continuous surjections  $T : C_p(X) \rightarrow C_p(Y)$ . Recently, in a joint paper with Eysen [1] we proved that the existence of such a surjection implies  $\dim Y = 0$  provided  $\dim X = 0$ . That answers a question of Kawamura-Leiderman [3] who established the same result for compact spaces. In the present paper we show this is also true for linear continuous surjections  $T : D_p(X) \rightarrow D_p(Y)$ , where all four possible cases are considered:  $D_p(X)$  is either  $C_p(X)$  or  $C_p^*(X)$  and  $D_p(Y)$  is either  $C_p(Y)$  or  $C_p^*(Y)$ .

Our result is obtained as a corollary of the following theorem:

*Let  $X$  and  $Y$  be Tychonoff spaces and  $T : D_p(X) \rightarrow D_p(Y)$  be a surjective continuous linear map such that  $|\text{supp}(y)| \leq m$  for every  $y \in Y$ . Then  $\dim X \leq m \cdot \dim Y$ .*

Similar estimation was established by Zakrzewski [5]: If  $X$  and  $Y$  are  $\sigma$ -compact spaces and  $T : D_p(X) \rightarrow D_p(Y)$  is a surjective continuous linear map such that  $|\text{supp}(y)| \leq m$  for every  $y \in Y$ , then  $\dim Y \leq m \cdot \dim X + m + m! - 1$ .

## References

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